THE FINANCIAL SURVIVAL PROBABILITY OF LIVING ANNUITANTS

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Abstract

This study addresses the question of how long a given amount of capital will be able to fund a living annuitant if the following five parameters are known: expected retirement duration (i.e. years between date of retirement and date of death), return on investment, inflation, annual withdrawal amount and initial capital amount available. A model (the Pension Model) that graphically depicts the relationship between these parameters was developed. This model facilitates retirement planning by showing how retirement duration and withdrawal rates change the financial "Survival Probability" (SP), which is the probability of having enough capital to maintain a desired withdrawal rate for the expected retirement duration. The underlying model is based on long-term historical investment yields of equities, bonds and cash in South Africa using Monte Carlo simulation with Cholesky factorisation.

Keywords

living annuity, money death, retirement duration, survival probability

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1. INTRODUCTION

This study addresses the question of how long a given amount of capital will be able to fund a living annuitant if the following five parameters are known: expected retirement duration (i.e. years between retirement and death), return on investment, inflation, annual withdrawal amount, and capital amount available.

A model (the Pension Model) that graphically depicts the relationship between the parameters listed above was developed. This allows for retirement planning by showing how retirement duration and withdrawal rates change the financial "Survival Probability" (SP), which is the probability of having enough capital to maintain a desired withdrawal rate for the expected retirement duration. Retirement duration is based on the Real Age-adjusted life expectancy which was developed in a previous article in this journal. The living annuitants can then decide whether they are comfortable with the financial SP of their chosen withdrawal rate, or whether their withdrawal rate should be reduced in order to increase the financial SP.

The Pension Model was based on a Monte Carlo simulation with Cholesky factorisation of historical trends in order to calculate the returns for various portfolios. Portfolios were classified as high-, medium- and low-risk. Ten thousand simulations were run for each scenario. Only the returns of equities, bonds and cash were considered in the model, with the figures being based on long-term historical data. It was assumed that retirees purchased a living annuity at age 65. It was also assumed that the living annuity was the only source of income during retirement.

In order to calculate what the financial SP was for a given age at death, the actual yearly benefit as a percentage of the funds available was compared to the desired benefit as specified by the living annuitant for each possible return scenario. Initial benefits were determined as a fixed percentage of the capital amount available with the withdrawal increasing in line with inflation. The financial SP is thus the number of iterations where the actual benefit received was at least equal to the desired benefit, divided by the total number of iterations.

Limitations on the minimum and maximum annual amounts that may be withdrawn from living annuities are also included in the Pension Model. These limitations are set out in the Association for Savings and Investment South Africa (ASISA) guidelines and impact the probabilities given the fact that they place limits on the amount of funds that can be withdrawn at any time.

The remainder of this paper is organised as follows. Section 2 briefly reviews the most important international and local literature published on this subject. Section 3 discusses the development of the Pension Model. Section 4 reports the results and Section 5 discusses conclusions.

2. LITERATURE REVIEW

During the past few decades there has been an on-going debate about the relative merits of life annuities (or annuitisation) and living annuities (or self-annuitisation) – both locally and internationally. Horneff, Mauer, Mitchell and Dus (2008) identified 22 international studies between 1965 and 2007 which compared the merits of these two approaches. A number of these studies also investigated the merits of combining the two. Two of the most prominent examples are the studies by Milevsky and Young (2002) and Dus, Maurer and Mitchell (2005).

While life annuities provide invaluable longevity insurance that cannot be replicated by pure investment vehicles such as living annuities, Goemans and Ncube (2008) point out that the popularity of living annuities in South Africa has risen significantly over the last decade. This may be due to the following factors:

- There is a loss of liquidity in the case of life annuities, as assets cannot be recovered after the annuity has been purchased (Albrecht & Maurer, 2002);
- There is no chance of bequeathing money to heirs, even in the case of an early death of the annuitant (Albrecht & Maurer, 2002). Rusconi (2006) pointed out that the bequest motive dominated the thinking of South Africans who selected an income drawdown strategy. However, according to a study conducted by Gardner and Wadsworth (2004), less than 40% of those United Kingdom (UK) retirees who chose not to annuitise were motivated by this reason;
- Gardner and Wadsworth (2004) also found that the main reason for retirees in the UK not annuitising was their desire for flexibility and the resulting control over their investment fund assets. This may well also be true in South Africa, where living annuities offer flexibility in terms of asset allocation, underlying investments and to some extent income drawdown rates (Goemans & Ncube, 2008);
- Rusconi (2006) suggests that the level of interest rates in the economy plays an important role in a retiree's decision to annuitise or not. The poor demand for life annuities in South Africa in recent years may well be due to the low level of interest rates, particularly real interest rates;
- Goemans and Ncube (2008) indicate that retirees choosing not to annuitise believe that they can earn a risk premium above the matched returns provided by life annuities. The long period of strong economic growth and the concomitant rise in share prices in recent years have created the illusion of wealth, which explains the low demand for life annuities in South Africa (Rusconi, 2006);
- Commission rates for life annuities in South Africa are not high. Financial planners therefore have a strong incentive to recommend instead the income drawdown model with its potential of more substantial on-going financial reward for the financial planner (Rusconi, 2006). This also seems to be the case in the market in the United States (US) (Reno, Graetz, Apfel, Lavery & Hill, 2005);
- Rusconi (2006) also states that falling levels of trust in South African insurers have contributed to individuals' reluctance to annuitise. This is echoed by Gardner and Wadsworth (2004) in relation to the UK market;
- Additional sources of income from government via inflation-linked social security or old-age grants may lessen the need for living annuities in first world countries (Horneff, Maurer & Stamos, 2008). This applies to South Africa to a very limited extent with its very modest old-age pension;
- The presence of a "family strategy", as posed by Schmeiser and Post (2005), may support the purchase of a living annuity. According to this strategy heirs are willing to bear the shortfall risk of the retiree's self-annuitisation, since they may benefit from a bequest. It is highly debatable whether this factor applies to South Africa.

In view of the factors outlined above this study will focus on living annuities in South Africa.

Goemans and Ncube (2008) did an extensive analysis of the main types of annuities in South Africa by making use of Monte Carlo simulation. Living annuities were compared with level life annuities increasing annually by 0% and 5% respectively, as well as a partially guaranteed life annuity, namely a with-profits annuity. The authors found that the two life annuities and the with-profits annuity were far superior to living annuities on a risk-value basis. They also concluded that, should a living annuity be selected, an initial drawdown rate of 2.5% to 5% is advisable if an income is required which grows with inflation. Finally, Goemans and Ncube (2008) concluded that it is prudent to have a maximum of 25% invested in equities in the case of a living annuity.

As was the case in the Goemans and Ncube (2008) study, this study is also based on Monte Carlo simulations. However, the point of departure in this study is that a living annuity was selected for the reasons listed above and that the Real Age-adjusted life expectancy of the annuitant has already been established.

3. PENSION MODEL

This section covers the development of the Pension Model for living annuitants in which the financial SP, the probability that the annuitant will have sufficient funds at date of death, is calculated.

3.1 Security returns and asset allocation

Firer and McLeod (1999) determined the performance of equities, bonds, cash and inflation in South Africa for the period 1925 to 1998. This data set was updated to 2010 by Firer (2011) and forms the basis of this study. The returns and standard deviations calculated from the Firer (2011) data are summarised in TABLE 1.

TABLE 1: Summary of returns and standard deviations for the asset classes

Asset class	*Mean return*	*Standard deviation*
Equities	15,6	19,5
Bonds	8,4	7,0
Cash	8,3	1,6

Source: *Firer (2011)*

For the purpose of this study three risk profiles are assumed. Risk profiles are determined by the equity exposure of each portfolio, as stipulated below.

- High-risk: 75% in equities
- Medium-risk: 50% in equities
- Low-risk: 25% in equities

The cash component in the portfolio varies and is assumed to be equal to the annual withdrawal rate (typically 2.5% to 10%). It is assumed that the balance of the non-equity component is invested in bonds.

3.2 Monte Carlo simulation with Cholesky Factorisation

Returns on various assets classes are stochastic in nature, making future prediction difficult. The starting point of the modelling was thus first to look at historical data and determine the average returns of the markets. These returns and the standard deviations of the returns form the basis of the Monte Carlo (MC) simulation. Ten thousand random samples were used to ensure sufficient iterations have taken place.

Further analysis of the historical data compiled by Firer (2011) showed a correlation between the various asset classes. Given the correlation, it would be incorrect to assume that the asset classes were independent and to run a straightforward MC simulation. Cholesky factorisation (CF) was introduced to create the correlation between the data. CF is used to generate x number of value sets that are described by a certain correlation matrix (Haugh, 2004).

Simulations were done on a yearly basis, with the benefit withdrawn at the beginning of the year and the investment return accrued at the end. The annual benefit withdrawn is explained in section 3.3.

Simulations resulted in a probability of survival (SP) for each age after retirement starting at 66, therefore assuming that the retirement age is 65. The probability was determined by calculating the number of failures during each of the ten thousand simulations. A failure was noted when the actual benefit received, for whatever reason, was less than the desired benefit. The cut-off age in the simulator is 100, as there are few people who live beyond that age. The model is, however, adaptable should older ages be required.

3.3 Withdrawal rates

A fixed withdrawal plan, which is a function of the annual fund value, was used for the model, albeit only one of a number of plans available to retirees. The simulations were run with 2.5%, 5%, 7.5% and 10% withdrawal rates. These rates were chosen on the basis that in practice withdrawal is typically 2.5% – 10% of the fund value. The withdrawal rate is used to determine the initial benefit received based on the pension fund value on the day of retirement. It was assumed that the withdrawal increased in line with inflation, which was assumed to be 6% based on the Firer (2011) data.

There are a number of considerations when withdrawing from a living annuity as stipulated by ASISA. One is particularly pertinent to the model: a benefit may only be withdrawn if it is between 2.5% and 17.5% of the fund value of the living annuity. If the required benefit is less than the stipulated 2.5%, the full 2.5% must be withdrawn from the fund. This does not result in money death (MD), because the amount received was still more than the required amount. Similarly, no more than 17.5% may be withdrawn during a year. Given that the 17.5% is an upper limit, it would be the second most likely cause of MD, apart from running out of funds. The term “money death” has recently been developed in the US and refers to the risk that a pensioner may run out of money in retirement.

3.4 Management fees and tax

Management fees for the various portfolios were determined according to the weights in each asset class. Performance fees have been excluded. Fee structures were obtained from the large investment houses, such as Allan Gray and Old Mutual. The average costs of managing an equity and bond portfolio are 1.25% and 0.77%, respectively. The cost of holding cash was based on the average fees of money market funds. The cost was determined to be 0.4% pa.

Tax is excluded in this study. All the required withdrawal amounts are pre-tax figures.

4. RESULTS

This section presents the results obtained from the Pension Model. They are presented using the survival probability concept in a form that is understandable by someone without a financial background. If reference is made to the probability of MD, it is the complement of the SP (see Equation 1).

$$SP\ (\%) = 100\% - MD\ (\%) \tag{1}$$

4.1 Pension model results

The results are presented in graphical format for each of the three risk profiles such that the financial SP could be read from it. FIGURE 1 to FIGURE 3 show the financial SPs for a given age and withdrawal rate for low-, medium- and high-risk profiles, respectively. The lines in the figures represent financial SPs expressed as a percentage. As is expected, the higher the withdrawal rate and the longer the retirement period, the lower the financial SP – regardless of risk profile. However, it is interesting to note that if a relatively high withdrawal rate and a long retirement period are required, then the financial SP is higher in the case of a high-risk portfolio compared with a low-risk portfolio. For example, if a 7.5% withdrawal rate and a terminal age of 90 are required, the financial SP in the case of a low-risk portfolio is 32%, compared with a financial SP of 49% in the case of a high-risk portfolio. Conversely, if a low withdrawal rate and a relatively short retirement period are required, the financial SP is higher in the case of a low-risk portfolio. For example, if a 2.5% withdrawal rate and a terminal age of 80 are required, the financial SP in the case of a low-risk portfolio is 83% compared with a financial SP of 77% in the case of a high-risk portfolio.

It is evident that if a high withdrawal rate and a long retirement period are required, then the annuitant has to accept higher risks and thus higher volatility in order to achieve higher returns. It is also evident that the financial SPs in the second example above are significantly higher than in the first example, as expected.

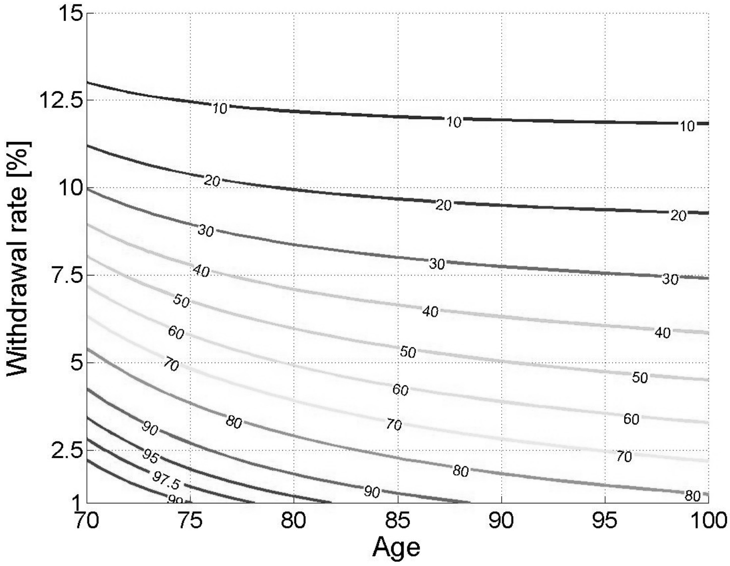

FIGURE 1: Financial survival probability for the low-risk portfolio

Source: *Authors*

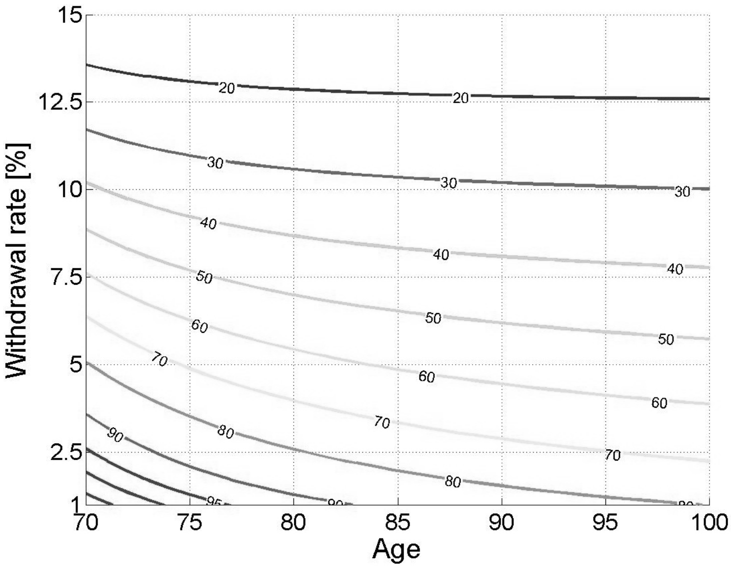

FIGURE 2: Financial survival probability for the medium-risk portfolio

Source: *Authors*

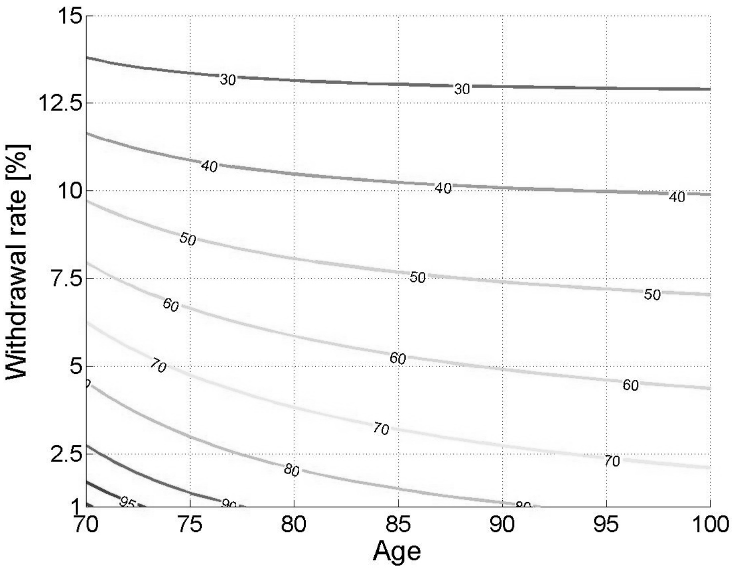

FIGURE 3: Financial survival probability for the high-risk portfolio

Source: *Authors*

4.2 Financial survival probability as a function of risk profile

Inspection of FIGURE 1 to FIGURE 3 showed that a common cross-over point existed where it was immaterial whether a low-risk portfolio or a high-risk portfolio was selected. To investigate this cross-over point, three financial SPs (50%, 70% and 90%) were plotted for the various portfolios on one graph in FIGURE 4. From this it can be seen that the cross-over point of all three portfolios is the line of a financial SP of 70%, because along this line all the portfolios essentially have the same risk for any withdrawal rate. This means that when a retiree chooses a financial SP of 70%, it does not matter in which portfolio he or she invests, irrespective of the terminal age and withdrawal rate chosen.

As the financial SP desired drops below 70%, the retiree should invest in a high-risk portfolio, again irrespective of age and withdrawal amount. The converse is true for higher financial SPs. If the financial SP desired is above 70%, then a low risk-portfolio should be chosen. The terminal age and withdrawal amount are thus independent and the portfolio could be chosen based solely on the financial SP.

This unexpectedly simple result is an elegant step for retirement planning in that it should ensure that the individual understands the concept of financial SP and then bases further decisions on it by considering the generic graphs.

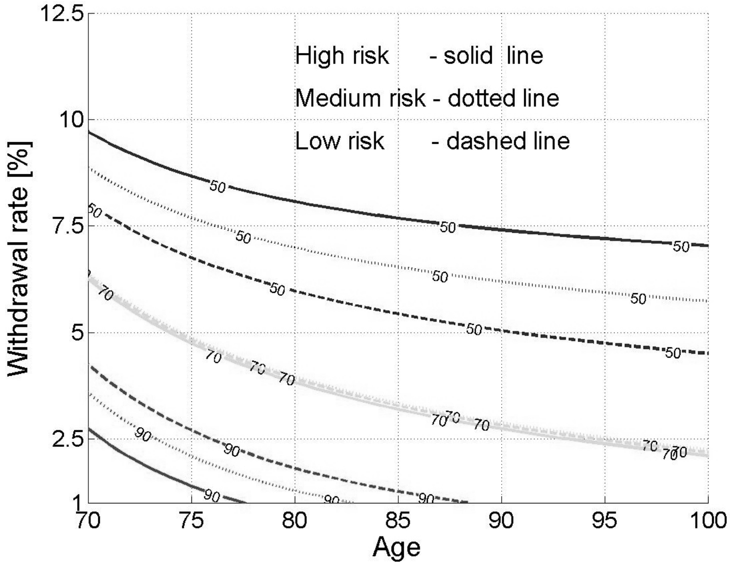

FIGURE 4: Survival probabilities as a function of time and withdrawal rate

Source: *Authors*

4.3 Impact of volatility

In the previous section it was seen that low-risk portfolios generally perform better than high-risk portfolios for retirees wanting more than a 70% chance that the portfolio will offer the desired benefits. This was attributed to volatility in the portfolio, though no comprehensive explanation was given. To explain this, MATLAB, a high-level language and interactive environment for numerical computation and programming, was used to generate one million random samples for each portfolio to determine the number of failures for each withdrawal percentage.

Failures (or MD occurrences) were plotted against the withdrawal percentages. In this way, the total number of failures can be summed to determine the overall probability of not reaching the desired withdrawal rate. A 2.5% withdrawal rate is assumed and is displayed by a solid vertical line on each graph. The failure count (or area) to the left of the line thus represents the probability of MD. In FIGURE 5 there are roughly 125 000 failures at the 2.5% withdrawal rate. Similarly, where FIGURE 6 and FIGURE 7 display the medium- and high-risk portfolios, the failure counts are 177 500 and 208 000, respectively.

It can thus clearly be seen that in a high-risk portfolio there are many more failure occurrences when compared to medium- or low-risk portfolios. Movement in the equity market causes the large volatility, which means there are many more times that the desired returns are not met or are even lower than that of lower risk portfolios.

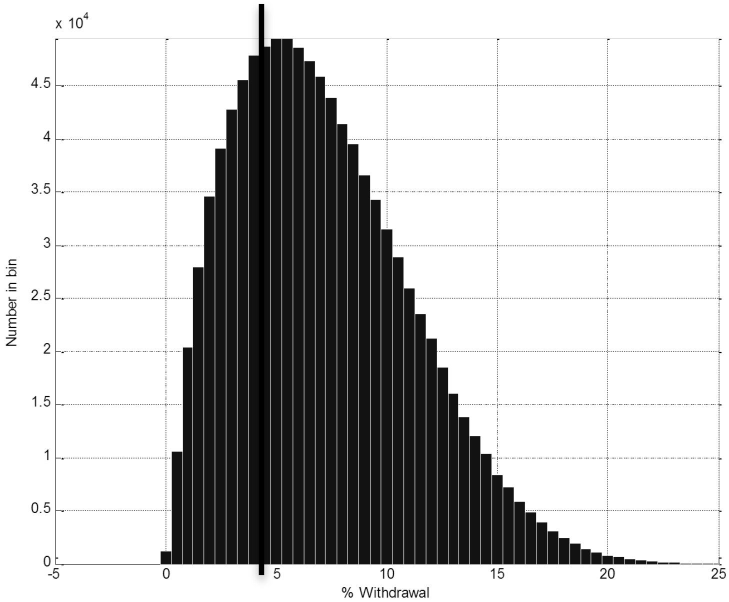

FIGURE 5: Failure rates for low-risk portfolio

Source: *Authors*

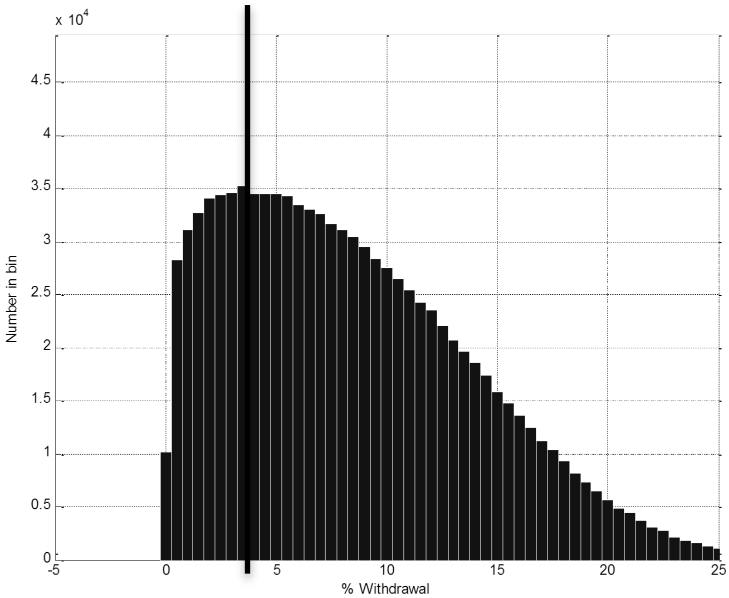

FIGURE 6: Failure rates for medium-risk portfolio

Source: *Authors*

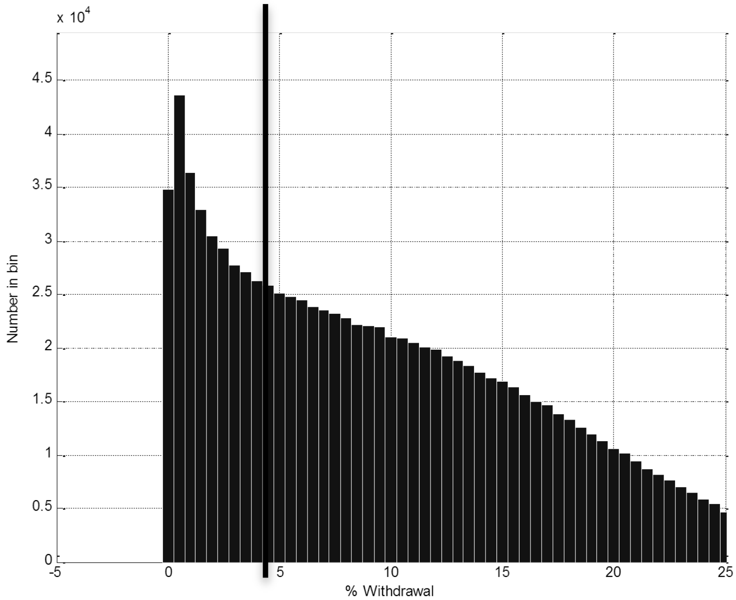

FIGURE 7: Failure rates for high-risk portfolio

Source: *Authors*

5. CONCLUSIONS

A Pension Model was developed which enables living annuitants to calculate their financial survival probability (SP) for a given withdrawal rate, risk profile and expected retirement duration. This model was based on Monte Carlo simulation with Cholesky factorisation in order to calculate returns of various portfolios. Portfolios were classified as high-, medium- and low-risk.

The following conclusions can be drawn from the development of the Pension Model:

- If a high withdrawal rate and a long retirement period are required, then the financial SP of a high-risk portfolio exceeds the financial SP of a low-risk portfolio. In this case the annuitant has to accept higher risks in order to achieve higher returns;
- Conversely, if a low withdrawal rate and a relatively short retirement period are required, then a low-risk portfolio offers a higher financial SP. However, in this case the financial SP is significantly higher than in the former case, as expected;
- A common cross-over point exists where it is immaterial whether a low-risk portfolio or a high-risk portfolio is selected. This cross-over point is a financial SP of 70%. As the financial SP desired drops below 70%, the retiree should invest in a high-risk portfolio, irrespective of retirement duration and withdrawal rate. The converse is true for higher financial SPs;
- The volatility of expected returns of the three risk-profiled portfolios has a significant impact on the probability of money death (MD). If a withdrawal rate of 2.5% is

assumed, the probability of MD is 12.50%, 17.75% and 20.80% respectively in the case of the low-, medium- and high-risk portfolios.

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